The Linear Etruscan Measurements of architecture

Abstract
To date the measurements of Etruscan architecture have remained unknown. The author has carried out a long study on the pre-Pythagorean mathematical language of the "Project of architecture, town and territory" (from now on "Project") in ancient civilisations. He presents -as one of the results of this- an in-depth investigation of the Etruscan Project, starting from the discovery of the measurements, which were part of this language. The analysis of the architecture and the towns gives new voice to the Projects, after 2500 years. The study is entitled: *The Search for E: the Project of Architecture, Town and Territory in Etruria and the Ancient World.*

Foreword
Etruscan archaeology has not so far yielded any objects identifiable as units of measurement, which we have instead for example for the Egyptians and Sumerians. Latin literature makes rare, vague allusions to Etruscan measurements. Some information has come down to us from the *Corpus Agrimensorum Romanorum* (Latin land surveying books of the Imperial Age). These cite for example the Latin *Actus*, specifying its ancient, probably Etruscan root, *Acnua*. There must have been the *foot*, because all the contemporary civilisations had the *foot* among their units of measurement; and maybe also the *passus (*'pace'*),* i.e. the 1 x 5 multiple of the *foot,* also maintained in the Roman age.
None of archaeologists' attempts at attributing a unit of measurement to the Etruscan monuments has yielded any useful results, and the findings of the sites are mainly expressed in measurements of the decimal metric system. The unit of measurement most tested to date has been the *Attic foot*, since, as is known, experts suggest a particular Greek influence on Etruscan culture. The *Oscan foot* and *Italic foot* have also been proposed from time to time because their measurements in centimetres vary with regard to the *Attic foot* and seem more appropriate for some but not all of the monuments. The literary approach to measurement has failed.
Applying a single measurement of about 27-30 cm such as the foot, without submultiples and multiples, cannot however lead to any useful result. What we need is a *system of measurement*.
I believe that research should have followed different, mathematics-based methodologies.
The Etruscans built stone monuments which go back to the 7th-6th CBCE, and in this age strong oriental traits can be recognised. All the temples and tombs present archetypical geometrical forms such as squares, circles, triangles, rectangles, cubes, cylinders, half-spheres; and there is a markedly symmetrical relationship among the different parts. This means that there must have been a *Project* with construction rules, as also appeared evident to Vitruvius, who codified a *Tuscan Order* for the Etruscan temples. In the first pre-Pythagorean Etruscan centuries, the mathematical knowledge of the priests/architects might have been that present above all on the Asian coast of the Aegean, originating from the great ancient civilisations of Mesopotamia and Egypt, and from Canaan. It was in fact from here that the new alphabet set sail for Mediterranean shores.
During my study I have reconstructed part of the pre-Pythagorean mathematical language applied to ancient architecture, which I have called "*Mathematics of the Origins*", with its geometrical and arithmetical contents, attributing to it a value that the term "pre-Pythagorean" did not allow it. In this mathematical system we find some fundamental theorems such as the ratio between the square and the circle, expressed in Egypt already in the third millennium by the *whole numbers* 5,7,22 (5: side of the square; 7: diagonal of the square and diameter of the circumscribed circle; 22: circumference of that circle), as contained in the measurement system of the Royal Cubit, which thus reveals a rigidly mathematical origin: the RC is the diagonal-diameter divided in 7 Palms; 5 is the RR, Royal Remen. The 22:7 ratio represents Pi. The 7:5 ratio represents the constant between the diagonal and the side of square (both in whole numbers). In Sumer, already in the 4th MBCE there was a system of surface measurements based on geometrical figures, notably including the fundamental one of the 1 x 2 rectangle (composed of two 1 x 1 squares). Finally, the ancient civilisations shared the same methodology of orientation based on the catheti of the right-angled triangle, hence on a pair of numbers. For example, a 3:4 orientation referred to the 3,4,5 Pythagorean triangle, considered sacred and dedicated to Isis in Egypt.

I mention these themes because the measurements were significant for the *numbers* which identified them. Hence, knowledge of the measurements is not an almost useless accessory for understanding an ancient monument (as is commonly believed), but a fundamental tool for understanding the *language* of the *Project*.
I cannot go further into the question here; I simply wish to observe that symbolic mathematical language, originating in the Egyptian and Mesopotamian civilisations together, permeated the ancient world. We find it in its entirety again in the Etruscan civilisation in Italy. Nor does it not end with them.
It would thus seem to me coherent, as a research theory, to attribute to the Etruscan civilisation, who used similar languages to those of the cultures present on the Asian coast, the same knowledge as is contained in the *Mathematics of the Origins*. I will not dwell here on the problem of how, in what circumstances and where they acquired it, merely acknowledging that they did.

The Theory

The theory I intend to demonstrate is that the architecture, the town and the territory of the Etruscans were planned with a symbolic geometrical/numeral language deriving from the *Mathematics of the Origins*. And that, by means of this language, we can today identify the units of measurement used by the Etruscan priests/architects. And finally, that the mathematical language and the numbers of the measurements together offer a deeper knowledge not only of their architecture but also of their culture.

Methodology of analysis

The transformation into *feet (pes)* of the metrical measurements of a monument almost never provides clear, unequivocal data, both because of the difficulty in understanding the ancient measurement used and because sure measurements cannot be identified over the long distances, and no submultiples are known of over the short ones, below 30 cm approx. of the *pes*. It has been hypothesised that the *pes* could be divided by fractions, for example ¼ *pes*, 1/12 *pes* as for the Latin measurements *Palmus* and *Uncia*, which are later. However, the problem remains of why the long measurements too often are not multiples of the *pes*; or why one measurement of a monument can be expressed in whole numbers which are multiples of the pes, and another one from the same monument cannot. We thus need to find an *Etruscan System* of measurement, bearing in mind the constant goal of the ancients to measure lengths and surfaces in whole numbers.
In this research project we must also assess both the state of maintenance of the stone structures and the tolerance of the measurements in the transposition from the building plan, as still happens today. For this, we need to take multiple measurements and apply Statistics, both as weighted average and as frequency.
All this must have discouraged scholars, particularly considering the existence of a certain materialist type of conviction that knowledge of the original measurements of a monument is unimportant in order to understand it. It has been a serious mistake because, in the assessment of the Etruscan culture, the contribution deriving from the content of the *mathematical language* used has been left out. The fact that Vitruvius had used it to illustrate the features of the Etruscan Temple has been neglected; the *Etruscan fable* recorded by Pliny, the so-called "*Tomb of Porsenna"*, was also created with the same language. In the ancient world, it had the same importance as the word, because it expressed concepts.
Another fundamental element which has been neglected in the research into measurements has been Geometry, which in ancient mathematical language was closely connected to numbers. Thus the *Project* was born, made up of geometry and numbers which expressed concepts by means of *measurements* and *orientation*.
To find the Etruscan measurements, I turned precisely to the *Geometry* and *Arithmetic of the Origins*. For example, my methodology of analysis of a temple consists of dividing the site plan deriving from archaeological investigation into geometrical figures, starting with the general symmetry of the monument; then finding modules (which respect the wall divisions) which compose small square or rectangular grids for reading multiple numbers of compatible measurements; thus arriving at defining a system of units of measurement. That is to say, I use geometry to reduce the linear measurements to elements which are easier to study in small segments.
Besides geometrical figures, I used elements supplied by Vitruvius for the Tuscan temple, such as dimension 5 of the façade and 6 of the depth, and the division of the front into 10 parts and the 3-4-3 division of the cells.
In ancient architecture, *the Project came before the measurement*, and was based on principles such as the symmetry of the parts and the composition of elementary geometries. These figures were identified

by *Numbers*, which were primarily symbols and only secondarily spatial elements. All this formed a mathematical language, as the expression of the *Project.*

Ancient architects always reasoned according to arithmetic, numbers and geometry together, as a single discipline, i.e. mathematics. Hence I have applied interpretative methodologies which use identification of the flat geometrical figures of the *Mathematics of the Origins*. In this way, I have found the sure measurements of the ten most important temples of Etruria and of the many mound-tombs I have analysed.

I describe below how I identified the basic system of the linear measurements through the analysis of two monuments.

First study case: the Palace of Murlo at Poggio Civitate: the *pes* and the *passus*

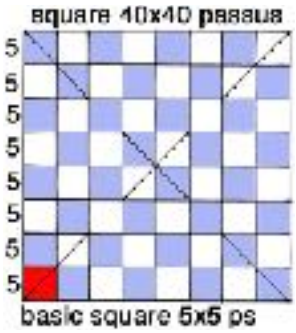
The palace of Murlo shows clear affinities with the mathematical system I have described above. It is an archaic 6th CBCE building. The date currently attributed to it is 575 BCE, which makes it contemporary with the *Heraion* of Samos, the first Ionian-style temple; and with the design of the Capitoline Temple in Rome. The ground-plan is a square of about 60 metres per side, with a spacious inner courtyard.

Adopting a possible measurement of the feet (*pes)* between 27 and 30 cm, the length of the side would go from 222,22 to 200 pes.

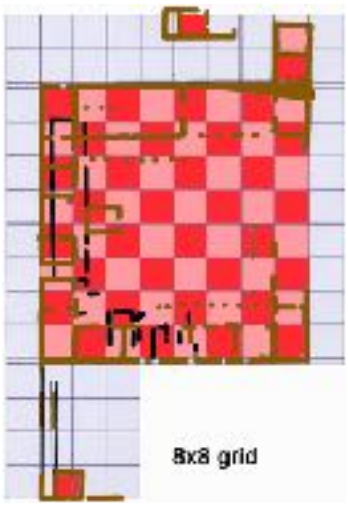
Let us now superimpose various grids with square boxes on the building. The first modular grid which fits the wall structures very well seems to be the one which covers the square with a chequer-board of 8 x 8 modules; 8 is known to be a number of the Etruscan *Templum*. The second grid is 16x16.

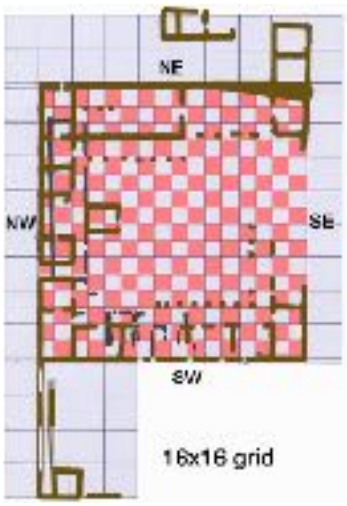
Each module will measure 60/8, i.e. about 7.5 metres per side. In pes, it varies according to the range adopted, from 27.77 to 25 pes.

Here it becomes clear that of the two measurements, that of 25 is more logical than 27 and 26 (whole numbers), if we remember that the *passus* (1 x 5) also comes into the pes system. In fact, our 25 *pes* can also be expressed as 5 *passus*. The total dimensions of the square of Poggio Civitate can thus be of 200x200 pes or 40x40 passus. The number 5 and its multiple 25 must have been a specific Project goal.



The number 5 recalls the archetypical side 5 square of the *Mathematics of the Origins*, of the system 5,7,22 square/circle which we often find in the Etruscan burial mounds.





Thus, not only is the *Form* an 8 x 8 chequerboard (perhaps a fully-fledged *Templum*) but each box is an archetypical square which bears principles of the mathematical universe represented by the numbers 5, 7, 22.

This result for the pes of the Palace of Murlo has been wholly confirmed by my study. I have verified that from the first burial mounds of the 7th CBCE to the temples of the 4th CBCE, and in all the Etruscan regions, the pes presents variations from 29.94 to 30.36 cm, with a range of difference of 4.2 mm and a mean value of 30.04 cm, which I normalise to 30 cm: from the large 7th CBCE burial mounds in the Arno Valley to the town of Marzabotto.

The pes and the passus do not satisfy all the monuments with readings in whole numbers, and here too they do not reveal all the measurements or the modular sub-grids which regulate the whole ground plan, including the courtyard.

A second case study: La Montagnola mound-tomb (Tumulus) in Quinto Fiorentino

The monument which really enabled me to understand the system of Etruscan linear measurements was the tholos tomb of La Montagnola at Quinto Fiorentino near Florence. It is one of the large tholos tombs of the Arno Valley, and is dated to the 7th CBCE.

I reproduce herewith the analysis of some of its chambers, exactly as I actually carried it out, so as to better follow the application and progression of the method and the reasoning.

The following is a list of the fundamental measurements:

Tumulus	approx. 70 m in diameter
Burial chamber	5.30 m in diameter
Vestibulum	1.75 x 6.85 m
Side chambers	1.50 x 3.00 m

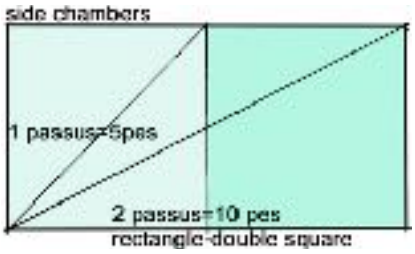
Of these measurements, the diameter of the tumulus is a datum not accepted by all scholars, but it is not an essential element for this study case.

The side chambers are a rectangle/double square

Let us begin with the simplest element. The ground-plans of the minor chambers are both rectangles formed by two squares:

1.5x3 m= (1.5x1.5)+(1.5x1.5). The measurements identify the most important Mesopotamian figure of the *Mathematics of the Origins*, which represents the same surface unit, the rectangle of 1 x 2 Kus (cubits). They are measurements which can be expressed in pes (of 30 cm): 5 x 10 pes or 1x2 passus.

The role of geometry is evident. The rectangle is equivalent to two 1x1 squares or two right-angled triangles of 1.2 catheti.

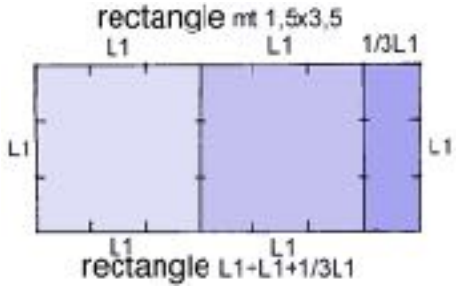


Now let us observe a decidedly important detail: according to *Columella* (one of the Latin authors of land surveying, who wrote in the 1st CACE), the rectangle of 5x10 pes corresponds to the smallest territorial surface measurement of the Roman *centurationes*, known as *"half a scripulum"*. He adds that it was an obsolete measurement in his time and very ancient: obviously Etruscan.

One might object that it is a question of chance. No, the whole body of Etruscan territorial measurements confirms that it is an Etruscan measurement used in the 7th CBCE in a very important tomb in the context of the *spatial division* of the Arno Valley.

The investigation into the system of measurements of the monument

The numbers in metres: 1.75, 6.85, the sides of the rectangle/vestibulum, to which I add the diameter of the burial chamber of 5.30, are so different from one another that they can only be justified by a complex system of measurements; thus if a system of Etruscan measurements exists, the resolution of this case can provide the solution to all possible cases.



Let us now tackle the problem of the measurement of 1.75, which is the width of the vestibulum. In the analysis, I helped myself out methodologically with its double, 3.50 m.

It may immediately be observed that there is no pes of 30 cm which fits the measurements of 1.50 and 1.75 or of 3.00 and 3.50 at the same time. That is to say, 1.75 and 3.50 cannot be expressed in whole pes. It is not the logic of the multiple series of the pes which can solve an interpretative problem of such dimensions. This example is a perfect expression of the dead end of metric research based on the pes. There must be something else. To understand the logic of the measurements we use geometric figures: referring to a typical rectangle such as the one formed by two squares, let us imagine one which has a side of 1.50 (1L) and one of 3.00 metres (1L+1L = 2L). If we wish to increase the long side to 3.50, the new measurement can be expressed according to the following equality: 3.50 = 1L+1L+1/3L. This very simple method of addition becomes particularly interesting if we attribute the measurement of 5 pes to the unit (1L= 5 like an archetypical square), because it is transformed as follows:

5+5+5/3=5+5+1.6(6)=11.6(6) pes. Thus we can state that the measurement of 3.50 metres corresponds to 11.6(6)pes, or 2 passus+1.6(6) pes, or 10 pes+5/3.

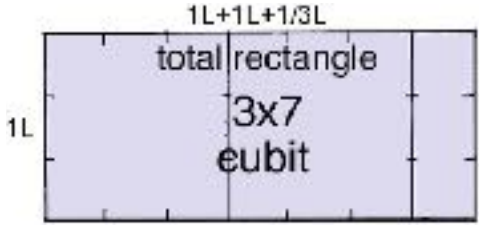
I wish to point out immediately that the ratio 5/3=1.6(6) is a special measurement that can be defined as a *proto-golden measurement*, in that it does not exactly supply the golden number 1.6(18…), but 1.6(66…). But this is not so important, because in any case it approximates, i.e. tends towards, the so-called golden number, inasmuch as the numbers 3 and 5 are the first significant terms of the famous succession of numbers known as the Fibonacci sequence (0,1,1,2,**3,5**,8,13,21,34,55,89,144,……), which presents the particularity that the ratio between two successive terms gives a number which *tends* to express 1,6180339887….., known in modern times as *Golden Number* or *Golden Ratio*.

The 5:3 ratio is perhaps the most significant (sacred) of the *Mathematics of the Origins*, and the main one in the mathematical language of the Etruscan monuments. Suffice it to remember that the Etruscan temple of Vitruvius is composed of two 5x3 (actually 5x6) parts; that many Etruscan altars contain these measurements/numbers, and that all the central cells of their tripartite temples have the 5x3 proportion.

We continue our analysis by observing that 1 passus is in turn 3 times 5/3 (5/3x3=5), since 1 passus= 5 pes, hence 5 pes = 5/3x3.

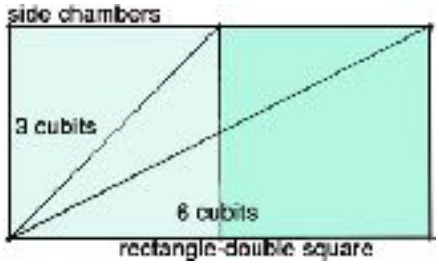
So the measurement of 2 passus (10 pes=3.00 m), which represents the long side of the rectangle of the side chambers, can be written thus: 6 times 5/3, i.e.: 2 passus= 6x5/3.

If we move on to the measurement of 3.50 m, it can be expressed by (6x5/3+5/3), i.e. 7x5/3. To understand better through decimal numbers, as we are accustomed to doing, the same formula can be written as (6x1.66) pes+1.66 pes= 7x1.66 pes. This expression leads us to retain that the metric basis of the measurement of 3.50 m is actually not the pes but the same measurement of 1.66 pes = 5/3 pes, which is the equivalent of 50 cm, the pes of this monument having been found to be equal to 30 cm. Thus the measurement of the long side of the rectangle constructed will be 7x(5/3) pes. The true basis of that measurement is expressed neither in passus nor in pes, but with a new measurement equivalent to 5/3 of pes. The sides of the rectangle measure: 3x5/3 pes on one side; and 7x5/3 pes on the other side. Because of the metrical closeness to the measurement of the *Cubit*, I define this measurement as *Etruscan cubit*; and because of the particularity of the number 5/3=1.6(6), we might also distinguish the Etruscan cubit as: *Gold Cubit* (GC) = 5/3 pes.



Returning to the side chambers

We have seen that the rectangle/double square measures 5x10 pes. In the double square of the side chambers we can now recognise the measurements in Etruscan cubits, which are 3x6 GC. They are significant archetypical numbers: the reading in pes (5x10) shows us numbers of Nature, of the real world; the reading in cubits (3x6) on the other hand presents numbers from the underground world. I observe that they are the same numerical classes that Vitruvius attributes to the ground-plan of the Tuscan Temple: 5x6 (composed of two 5x3 rectangles).



The result: The fundamental Etruscan measurement, the Cubit

Let us now specify the logic of the relationships between pes, passus and cubit, without forgetting that the two measurements of the vestibulum, 1.75 and 6.85 m, are still unknown, since they are expressed in neither pes nor cubits.

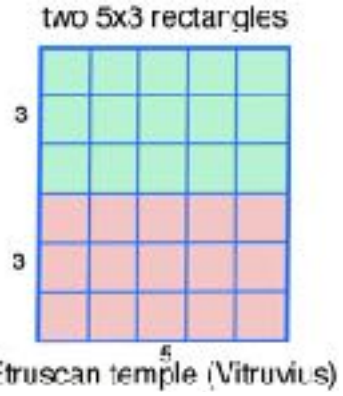
1 cubit (GC) = 5/3 Pes =1/3 Passus = approx. 50 cm
1 pes = 3/5 GC =1/5 Passus = approx. 30 cm
1 passus = 5 Pes = 3 Cubits = approx. 150 cm

The unit of measurement of the cubit can be found in all the Etruscan monuments I have examined (from the 7th to the 4th CBC), demonstrating that it was the main reference unit. We do not know its Etruscan name. The cubit aligns the Etruscan system of measurements with that of the ancient civilisations.

We should reflect on the presence of the pes and the cubit together in the Etruscan measurements, as happened in the ancient civilisations. They are harmonically complementary and together they provide a greater number of *whole numbers*. For the ancient peoples who developed the *Mathematics of the Origins*, this was very important, because much more *Nature* was thus assimilated with the *Universe*.
The metric result of the Etruscan cubit has been wholly confirmed in my study. I have verified that from the first mound-tombs of the 7th CBCE to the temples of the 4th CBCE, and in all the Etruscan regions, the cubit presents variations from 49.90 cm to 50.60 cm, with a range of difference of 7 mm and a mean value of 50.25 cm. The greatest frequency of cases is around 50 cm, which we might define as the metric measurement which is the equivalent of the mean Etruscan cubit. Thus we can claim the existence of units of measurement which tend to be uniform and permanent for the whole Etruscan nation, at least up to the 4th CBCE. The statistical result is very important because it demonstrates the geographical and cultural unity of this people in its mathematical language for at least 4 centuries. Probably the religious sphere which must have included the Project of Architecture enabled it to maintain its canonical precision for a long time.

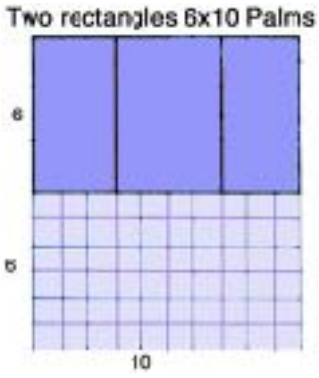
The key Etruscan measurement, the palm, is inserted into the Etruscan cubit system

Investigation of Etruscan measurements cannot stop at the discovery of a unit greater than the pes, i.e. the cubit, because this does not resolve the geometrical complexities which go beyond those I have used as examples, as is the case of the measurement of the cubit of the Tomb of La Montagnola. We must complete a *system of measurements* which is able to read sufficiently small ones. Logically, we need a new unit of measurement which must comply with the condition of being a submultiple of both the Etruscan cubit and the Etruscan pes.



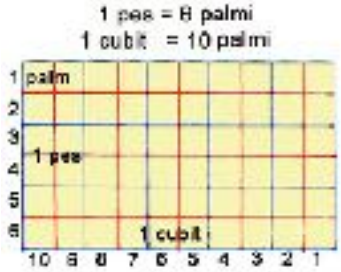
Etruscan temple (Vitruvius)

The result of this research has something ironical and mocking in it, because it has been staring all experts in the face for over 2500 years: it is to be found in the description that Vitruvius offers us of the Tuscan temple (Book IV, 7):
The area on which to build this type of temple must be a rectangle, in which the shorter sides (the width) must measure 5/6 of the longer sides (the length). This area will then be divided into two parts, the back area being reserved for the cell , and the front area for the colonnade. The width must also be divided into 10 parts, the last three of which on both right and left are for the smaller cells or, alternatively, for smaller structures, while the four central ones constitute the middle part of the temple.



Let us thus examine the 5x3 rectangle which makes up each of the two parts of the Tuscan temple (which is 5x6). The two measurements, 5 and 3, have no common divisors, since they are prime numbers. But if we double them, as Vitruvius points out, dividing the boxes in half geometrically, we obtain 6 boxes for the short side and 10 boxes for the long side.
This new geometry of the 6x10 *proto-golden rectangle* comes to express the reciprocal relationship between the Etruscan pes (short side of the rectangle=6) and the cubit (long side= 10). The length of the side of each box of the 6x10 rectangle expresses the *Etruscan Palm*, equal to 1/6 pes, 1/10 cubit. Thus the 6x10 rectangle is read as 1 pes x 1 cubit.
The measurement I have called *Etruscan Palm* allows the measurement of both the pes and the cubit to be shared, and this is extremely important, because it is the key to the architectural interpretation of the buildings.

That Vitruvian 5x3 rectangle, which becomes 10x6, expresses the three Etruscan measurements.
The relationship between the measurements is the following:
1 cubit = 10 palms =5/3 pes =1/3 passus = approx. 50 cm
1 pes = 6 palms =3/5 cubit =1/5 passus = approx. 30 cm
1 palm = 1/6 pes = 30cm/6 = = approx. 5 cm
1/10 cub= 50cm/10 = = approx. 5 cm



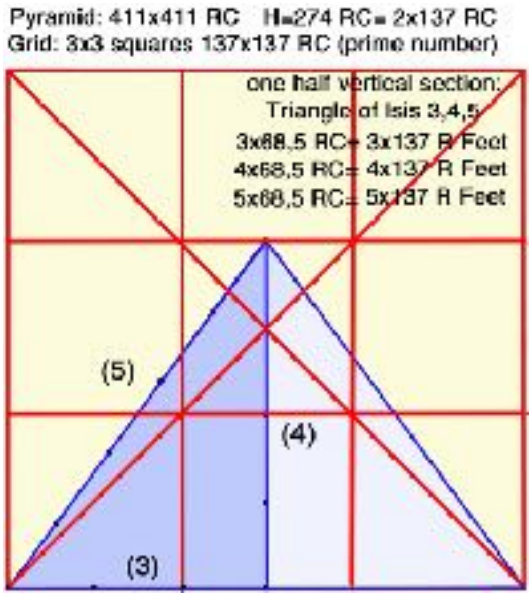
In metric terms, if we consider the mean cubit as approx. 50 cm and the mean pes as approx. 30 cm, the Etruscan palm will equal approx. 5 cm. In this way, we have found a sufficiently short measurement for guaranteeing better results in the analysis of the monuments.
The whole of my analysis of the buildings has been based on this small measurement which I have called palm because of the similarity to the subdivision of the Egyptian Royal Cubit, even if the two are not equivalent as measurements. I observe that the Romans too had a Palm, but in the ratio of ¼ of a Foot.
Perhaps the palm itself could be subdivided into *Fingers* on a decimal basis, each one equalling approx. 0.5 cm. But I do not retain this to be probable, since in the attempts I have made during my research on the temples and tumuli I have not found any readings of significant numbers, as instead happened with the palm.
The measurement of the palm could perhaps be split: for example 1/2 palm = 2.5 cm; or 1/3 palm = 1.66 cm, and so on; but this study has not found sure evidence for a conclusion to be reached regarding this aspect.
In the analysis of the monuments, every metric reading has been converted to the relative palm, using the methodology of conversion by approximation. The intermediate measurements of less than half a palm are expressed with the lower palm, and the ones over half with the higher palm.
Now we can also read the measurements of 1.75 and 6.85 m.

The numbers of the* vestibulum *1.75 and 6.85 m are transformed into 35 and 137 palms

The measurement of the short side of the vestibulum is expressed as follows:
1.75 m= 35 palms = 3 cubits + 5 palms = 5 pes + 5 palms.
They are three different readings each of which supplied a different mathematical language from the other. And one of them was chosen to state *concepts* through the meaning possessed by the *Whole Numbers*. In this case, the first reading, with 35, does not seem important (albeit characterised as 5x7); nor does the third reading of 5+5 (which can be interpreted as a strong allusion to Nature/Earth). For the choice of the expression of measurement, reference must also be made to the features of the monument, and the Tomb of La Montagnola is loaded with primary symbols, perhaps because the man buried there was an important personage. Thus I think the measurement would have been read as 3 cubits +5 palms to highlight the two fundamental numbers of the Etruscan world, 3 and 5.

The measurement of the long side of the *vestibulum* is expressed as follows:
6,85 m= 137 palms, prime number; = 13 cubits + 7 palms, prime numbers; = 22 pes + 5 palms.
Here we have three very interesting cases. The expression in cubits contains two fundamental prime numbers, 7 and 13. The measurement in pes contains the number of the circle/sky, 22, and that of the square/earth, 5 (of the system 5,7,22). But I retain that the fundamental number of the Project of the Tomb was the prime number 137, which in antiquity seems to have been very famous and particularly sacred. 137 was the measurement in Royal Cubits of the length of the Sphinx of Giza in Egypt. It was also the measurement of the grid of 3x3 square modules of 137x137 RC superimposed on the square of the Pyramid of Khafra with its ground-plan of 411x411 RC (411/3=137). The height of the pyramid was 274 RC, i.e. 2x137. The relationship between 411 and 274 means that 137 was a goal of the project. Studying the three Pyramids with the *Mathematics of the Origins* I have understood that in all three both Ra (the Sun) and Osiris (the night Sun) appear, and each one is identified by Numbers. In his book *Isis and Osiris*, Plutarch claimed that in Egypt the Gods were represented by Numbers, and the same applied to Greece. 137 was a reinforcement of 1 (Ra's main number), like 11 or 111; in fact 1+3+7 make precisely 11. And it is not by chance that 137 also characterises the Sphinx, as a solar symbol which exactly faces the East.
In this pyramid we find a *mathematical game* based on a significant exchange between Royal Cubit and Royal Foot to express whole numbers. The Royal Foot (little used) was exactly ½ of the other. It is found in the Pythagorean triangle 3,4,5 (called "Triangle of Isis" in Egypt), which forms the vertical half section of the pyramid.The three catheti expressed in RC are non-whole numbers. The triangle has a base equal to half of 411 (=205,5 RC), divided into 3 parts (=68.5 RC). The height of 274 Royal Cubits divided into 4 parts again gives the number 68.5. Side 5 will also have 5 parts of 68.5. These non-whole numbers of RC were not compatible. Thus the Royal Foot was adopted (2x68.5 = 137), which gave the numbers of the "Triangle of Isis" 3x137 RF, 4x137 RF, 5x137 RF, in order to have whole numbers.
Substantially, the number 137 distinguished both the pyramid of Khafra and the "Triangle of Isis", becoming the identifying number of that Pyramid.

With this reference I do not intend to claim that the Etruscans knew the three famous pyramids, only that non-coincidental number used by them must have been retained to be exceptionally sacred and probably dedicated to the Sun, or anyway to the chief of the Gods. In my study, I have found other numbers such as 137 which have been passed on millennia as sacred.



We must ask ourselves why we find such a sacred number in the *vestibulum*. My answer is that from its geometries I have found that the *vestibulum* must have been seen as the *Form* of the chthonian process of preparing the soul for its passage into the heavens, propitiated by certain Gods; in other words it was just as sacred an area as the burial chamber. But I cannot pursue this further here.

From these notes it may be understood just how rich the mathematical language of Etruscan architecture was and how deeply it was inserted into the *Mathematics of the Origins*. The re-interpretation of ancient and Etruscan architecture with this tool reveals the *Architectural Project* and its meaning, which differ from case to case.

Measurements do not end with numbers; their geometries must also be understood

Geometry was very important in the *Mathematics of the Origins*, together with Numbers.

It was the only element of the Universe which remained whole in its transfer to the Earth (but immaterial), while it was not so for all the numbers, for example for the irrational ones like Pi. This is why Plato spoke above all about geometry.

Geometry enables us to understand the case of the measurement of the diameter of the basic circle of the burial chamber of the Tomb of La Montagnola, which is 5.30 m, i.e. 106 palms=2x53 palms. We need to discover why a measurement based on the prime number 53 was adopted, considering that the *Project* is never random. The diameter is expressed as 2x53 palms.

The choice can be understood only after studying the geometries and numbers of many Etruscan Tumulus tombs and after discovering coincidences related to 53, which in the majority of cases refer to the diameter or radius of the Circle.

In the study of the mathematical language of the Project of the Etruscan Tumulus tombs , I have ascertained that the main and statistically most frequent elements regard three themes:

1 The search for prime numbers, which are by definition sacred;

2 The virtual transformation of the Square/Earth into the Circle/Sky, which symbolises the change of dimension of the Soul, represented both in the burial chambers and in the *vestibula* and tumuli;

3 The search for surfaces equivalent to geometric figures such as squares or rectangles, which are transformed into circles and vice versa, this being the tool for effecting the change of form from earthly to celestial. Let us verify if the area of the circle with the 2x53 Palm diameter can equal that of a regular polygon, for example a square. Let us use 53 and not 106. Pi is expressed with the fraction 22/7.

The surface of the circle is:

S_C= Pi r^2 = 22/7x702.25 = 2207 $palm^2$= LxL;

The side of the square will be L= $\sqrt{S_C}$;

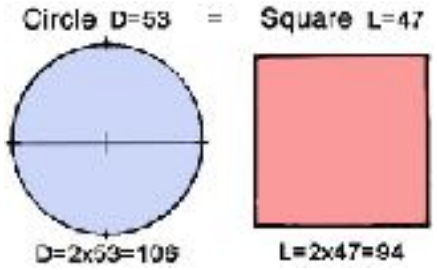
L= $\sqrt{2207}$ =46,97 =47 palms (the numerical values are approximated to whole numbers, as in the *Mathematics of the Origins*).

It did not take long to find an incredible result which can even be expressed as a rule: *"The surface of a circle of diameter 53 equals the surface of a square with sides of 47, where the numbers 47 and 53 are consecutive prime numbers in the series of prime numbers".*

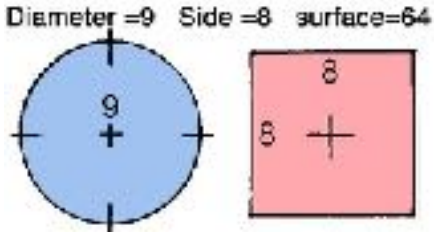
It is clear that the circle with a 2x53 diameter will be equal to a square with 2x47 sides.

Knowledge of the relationship between the circle of 53 and the square of 47 is testified to by other Etruscan monuments: for example the tomb of Monte Calvario has a measurement -at the drum of the tumulus- of 2x53 cubits. In the Populonia necropolis, the square burial chambers of the Tomb of the Carts and Tomb of the Funeral Beds measure 2x47 palms.

I wish to point out that it is always possible to construct one geometrical figure which has equivalent measurements to another (with the due approximations); hence the equivalence is demonstrated, i.e. knowingly designed, if the figures show particular numerical connections between them, as in the examples above, which involve prime and consecutive prime numbers.
The number forms a virtual figure, contained in the real one. If 47 is the measurement of the sides of the square, it does not correspond to the real measurement of the diameter of the dome, which is inscribed in the square. It means only that the measurement of the chamber makes it virtually earthly and celestial at the same time.



In order to understand the Etruscans' interest in the equivalence of surfaces between squares and circles, we need to refer back to the *Mathematics of the Origins* in Egypt, where problem 50 of the Rhind mathematical papyrus (XVIII CBCE) reveals its origin: *A circle with a diameter of 9 equals a square with sides of 8 and their surface is 64 (in whole numbers).* The origin of this theme is linked in Egypt to the *Judgment of Osiris*, which allows the passage of the soul from Earth to Heaven. In architecture, it is expressed in the transformation of the square/Earth into the circle/Sky. Think how many Christian churches have a square transept with an overhead dome; in those too the passage of the souls (of believers) from Earth to Sky takes concrete form. It is a figure which goes back at least 4.5 millennia.



I have used the new Etruscan measurements for both the architecture and the territory, discovering their efficacy with surprising results. In the end, the cubit and the palm have allowed me to re-interpret the dimensions of the main Etruscan temples, indeed to read in them for the first time after 2700-2400 years the geometrical arithmetical language which created them, i.e. to retrace the Project underlying them. At the same time, they have allowed me to interpret the farmland, the town and the necropolises, because the Etruscans had different (metrically) divided spaces for these works. The *Numbers* of the measurements and the *Geometrical Figures* were used to express symbolic concepts. What was important for these cultures was not the measurement interpreted non-religiously as we do (width, length, height), but the number which it expressed.
When Vitruvius states that the Tuscan temple is 6x5 (depth by width), he is not expressing a measurement, but *numbers* as the relationship among parts, which in this case also represent an eschatological concept, as I have interpreted it above. The number qualifying a measurement first and foremost expresses a symbolic concept. Seneca, who expressed the difference between Romans and Etruscans regarding lightning (*naturales quaestiones, 2.23*) might have said: *we believe that numbers are given to express measurements; the Etruscans instead believe that numbers are given to describe the cosmos.*
This is the importance of measurements in ancient architecture, including those of the Etruscan world.
We are talking about a fully-fledged *language,* because the Etruscan monuments speak, and they are in good company: the whole of the Egyptian and Mesopotamian world; the Hittites; the Phoenician-Canaanites; the Biblical world; the Minoans and Mycenaeans; then also Pythagoras and Plato.

The mean Etruscan linear measurements and the corresponding Roman measurements
I have combined the Etruscan and Roman linear measurements in a comparative table. We can see that, for the Romans, the base measurement became the foot, and that the cubit was nominally calculated as 1.5 times a foot, which is a simplification of the Etruscan ratio. There was a laicisation of the system.

Comparative Table of Etruscan and Roman Linear Measurements up to 3.00 mt.

Etruscan linear measurement	base: cubit=1	base: foot=1	normalised value cm		Roman linear measurement	base: foot= 1	normalised value cm

cubit	**1**	**1 X 10/6**	**50,00**		**cubitus**	**1 x 1,5**	**44,47**
foot	**1 X 6/10**	**1**	**30,00**		**pes**	**1**	**29,65**
palm	**1/10**	**1/6**	**5,00**		**palmus**	**1/4**	**7,41**
					uncia	**1/12**	**2,47**
					digitus	**1/16**	**1,85**
					gradus	**2,5**	**74,12**
passus	**3**	**5**	**150,00**		**passus**	**5**	**148,25**
pertica	**6**	**10**	**300,00**		**pertica**	**10**	**296,00**

The surface Etruscan Measurements

Analysis of the measurements of Roman spatial divisions

To find the Etruscan surface measurements we need to reconstruct the mathematics of the Roman spatial divisions, i. e. of their *centuriationes*. This was an intuition subsequently confirmed by the territorial analyses of Etruscan towns and territories, and partly suggested by Columella, one of the Roman men of letters who wrote about land surveying in 1CACE. He claimed that the linear measurement of the *actus* (200 *pes*) was called *acnua* in ancient times and that the smallest surface measurement was the half-*scripulum* (5x10 *pes*), no longer used in his time.

The Romans carried out a large number of territorial spatial divisions, especially during the imperial expansion, involving the whole of the Padana Valley from west to east, the Tuscan, Latium and Campania plains, many European river-valleys and African areas such as Carthage in Tunisia. Thus the Romans implemented the greatest transformation of the territory ever made before the modern age. But where did they learn these techniques?

Roman tradition deriving from various sources maintained that the geometrical spatial division of the territory according to orientated and measured Cartesian axes was of Etruscan origin. Sextus Iulius Frontinus, one of the preeminent Roman authors of the *Gromatici Veteres*, declared in the late I CACE " the first origin of the *Limitatio*, i. e. the spatial division of the territory mentioned by Varro, came from the "*Etruscan Disciplina*", which was part of the Etruscans' sacred books.

The Etruscans indeed did possess the arithmetical, geometrical and technological knowledge to carry out such divisions; and in the regions they inhabited there are many traces which can unequivocally be attributed to their works: towns, necropolises, agricultural systems, complex road networks, land drainage systems.

So let us look now at the theory and methodology of spatial division in order to understand the meaning of the numbers and geometrical figures it proposes. Let us examine the mathematical elements, starting from its fundamental figure, the large *elementary territorial square*, ETS, of 2400x2400 *pes* (approx. 712 m in Roman *feet*). I will group all the operations of land division (*limitatio*), boundary stone placement (*terminatio*) or lot distribution (*centuriatio*) under the definition *spatial division of the territory* (understood as geometrical).

The typical figure of territorial division was a square

The choice of the geometrical figure of the square first needs to be reflected on, in order to remind us of some of the concepts of the *Mathematics of the Origins*. In all parts of the ancient world, it symbolised Earth itself as a mirror of the Sky. It is the form of the man-made Earth which complies with the concept of Harmony. Thus it acquired a certain sacredness, which also included the protection of the boundaries of property. For example, marks of divinity were placed at the intersections of the lines of territorial division, such as crossroads. These were later transformed into shrines, where saints and madonnas appeared from the Middle Ages onwards. If we look today for evidence of ancient spatial divisions, the presence of repeated crossroads marked by shrines is often an indication of an ancient ritual division of the lands.

***The measurements of the Roman* centuriationes**

I propose to investigate, through the analysis of its measurements, whether the Roman ETS can qualify as Etruscan by virtue of elements which can undeniably be attributed to the Etruscan world; and whether the principles of the *Mathematics of the Origins* can be identified in it.

From the Roman *Agrimensores texts* (of the imperial age) we know that the units of measurement used by the Roman land surveyors were the *pes*, the *passus* and the *actus*, and that the *actus* was declared to be the most ancient measurement. So I decided I had to try and verify whether the measurements of the *pes* (unit=1), of the *passus* (1x5) and the *actus* (1x120) could be defined as Etruscan units of measurement, regardless of their metric value (in my analyses the mean Etruscan pes came out as slightly longer than the Roman one). We know that the tracing of a spatial division was a kind of rite, i. e. it had a religious context. On the territory, sacredness is a condition which tends to confer permanence on its main structures, hence it may be presumed that the same measurements were maintained in the practice of land surveying even when this activity had lost its religious significance and primitive contents, as is the case with Roman land surveying.

We note that of the three basic measurements, *pes, passus* and *actus*, the *passus* (1x5) is a decimal multiple, while the *actus* (1x120) is an abundant number (its proper divisors in the first set of ten are 1,2,3,4,6,8,10).

My analysis is presented with the same logical development that it actually followed. The first part of it occurred before discovering the Etruscan measurement of the *cubit* and the *palm*. By inserting the new unit of measurement after reaching a certain level of conclusions (as in fact happened), we observe how it widens the horizons of research to the point of allowing identification of the Etruscan geometries and measurements and, immediately afterwards, discovery of their sources.

So let us start from the ETS, the equivalent of the Roman *centuria*, and let us ask ourselves:

Why is an ETS made up of 2400* pes *(480* passus*, 20* actus*)?

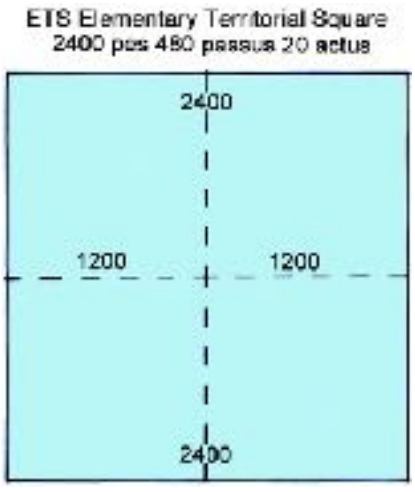
In my studies I have found no critical motivations for the ETS side measurements expressed with the numbers 2400, 480, 20. To me they seemed particularly to refer to a decimal construction of Etruscan/ Roman mathematical language, so the problem of finding an explanation for this measurement was one of the first that I posed myself, even before discovering the Etruscan measurements.

If the decimal system had been strictly applied, the square would logically have been divided on a basis of 10, as in the decimal metric system, where a hectare strictly consists of 100x100 metres with a surface of 10,000 square metres. The same might have happened with the *pes*; a linear measurement of 100 *pes*, a surface of 10,000 square *pes*.

It seems to me that the wish to fix a measurement with the number 2400 must mainly depend on the possibility of having a greater number of divisors than for example with 1000. The concept is peculiare to the *Mathematica of the Origins*, i. e. of the old civilisations.

A first analysis of the square form

Before proceeding any further, let us stop and analyse the presence of the *actus* among the Roman measurements, since it is in fact a different type of measurement from the others. *Actus* is the Roman name but the more ancient name *acnua* has come down to us and is probably in fact the Etruscan name.



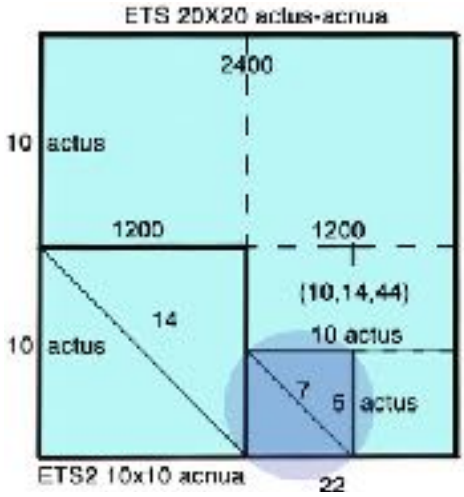
The square to be investigated is not the ETS (the Roman *centuria*) but the square whose side is halved, which I will call ETS2, and which is the smallest one with the same divisors as the other one. I will go further into this question later, limiting myself to saying here that the ETS2 of 1200 *pes*, 240 *passus* and 10 *actus-acnua* was the basis of a previous spatial division which the Romans remodelled with the ETS.

So let us investigate the elementary ETS2 square, starting precisely from its measurement of 10 *actus/acnua.*

We find this figure in the Egyptian version of the *Mathematics of the Origins,* in the context of the *royal cubit system* which originates from the *square/circle* ratio 5,7,22 (where 5 is the side of the square, 7 is its

diagonal and the diameter of the circumscribed circle, and 22 its circumference). The *whole numbers* deriving from this geometrical construction were expressed in *royal palms*, where 7 RP = 1 *royal cubit* and 5 RP = 1 *remen*. Another measurement of 10 *palms* exists, the *double remen*. In the development of that very old mathematical system, 10 is the measurement of the side of the square 5 doubled. This larger square has a diagonal in *whole numbers* of 14 and a new circumference of 44. Well, these numbers in *actus/acnua* are those of the ETS2:

- side of 10 *actus/acnua*
- diagonal=diameter of the circumscribed circle of 14 *actus/acnua* in *whole numbers*
- virtual circumference =44 *actus/acnua* in *whole numbers*



So the numbers of the Square ETS2, with a side of 10 *actus/acnua*, equal those of the *Archetypical Square* of the *Mathematics of the Origins*. Why?

Spatial division was carried out according to a religious ritual, whereby the terrestrial *Centurial Square* repeated the *Celestial Circle*, according to the mythological principle “on earth as in heaven”.

The presence of the *acnua* as a measurement of the centurial square, and the coincidence with the numbers of the *double royal remen*, indicate that it is sacred. It testifies the great age of the intellectual concept behind the centurial square ETS2 and, by extension, the ETS. The religious value of the *actus/acnua* caused it to remain the main measurement of spatial division also in the imperial Roman age.

In search of the ETS divisors

The numbers 2400, 480 are divisible by 2,3,4,5,6,8,10 in the first ten numbers (except for 1), and also for 12, 15 and 16 in the first 20, i. e. for 10 numbers altogether in the first 20. This fact certainly determined a deliberate choice.

If the numbers adopted had been the couple 2000, 400, the number of whole divisors in the first 20 would have dramatically decreased to 6 (2,4,5,8,10,16); and if it had been 100, it would have decreased to just 5 divisors. That is to say, passing from the number 1000 to the number 2400, the whole divisors in the first 20 numbers double from 5 to 10 (excluding 1). The difference between the pairs 200/400 and 2400/480 consists in the fact that the latter also contains the divisors 3,6,12,15, and in particular class 3 (excepting the number 9 in both cases).

Dividing the numbers of the pair 2400/480 by the 10 resulting divisors, the results are, respectively : 1200, 800, 600, 480, 400, 300, 240, 200, 160, 150, for the measurement of the *pes*; and 240, 160, 120, 96, 80, 60, 48, 40, 32, 30, for the *passus*.

The coincidence of 10 whole results simultaneously for 2400 and 480 cannot be due to chance, and it shows that *pes* and *passus* (i. e. 1 and 5) are inseparable as basic measurements of the ETS. The relationship of *pes* and *passus* to *actus/acnua,* characterised by the archetypical 20, shows that the three measurements form a system, and this system is ancient. For the time being I am excluding the *actus/acnua* from the analysis, because I intend to analyse this measurement in depth after the *pes*. The number 20 is divisible only by 2,4,5,10.

The halving of the ETS side: the ETS2

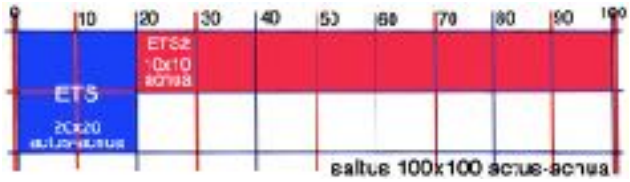
Halving the side of the ETS we obtain the theme 1200, 240, 10, which again allows us to maintain the joint divisibility of *pes, passus* and *actus*, with the *actus/acnua* brought significantly to 10.

In fact the ETS-*Centuria* is always subdivided into 4 ETS2 squares of 10 *actus/acnua* by the *Quintari.* These are important streets or streets-plus-canals, or streets-plus-ditches, situated at a distance of 10 *actus/acnua* (1200 *pes*; 240 *passus* in fact) from one another, corresponding to approximately 360 metres. This implicitly affirms the existence of the first certain divisor of the ETS, which consists of the number 2. No further halving of the ETS2 exists with the

numbers 600, 240, 1, which would actually be mathematically possible.
My analyses on the territories suggest that the ETS2 formed by the *Quintari* was the basis of the ancient spatial division which the Romans quadrupled, as I have mentioned above. It was certainly Etruscan. This is also confirmed by the composition of the *saltus*.

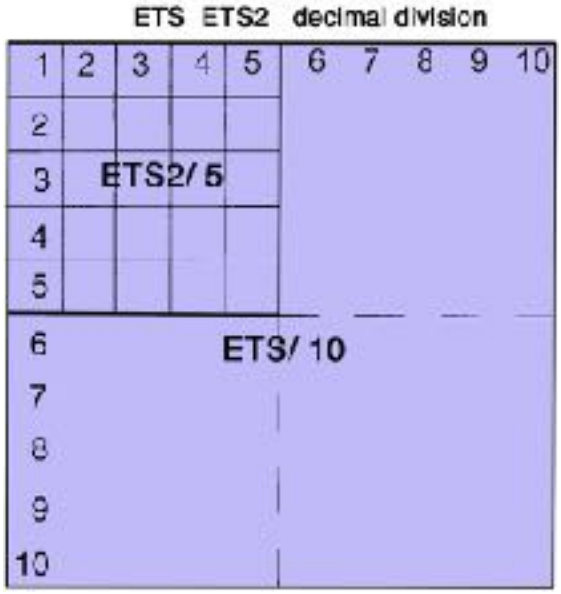
A recurrent multiple: the saltus

Another recurrent measurement in large-scale land surveying is the *saltus*, a multiple of ETS2, which equals 10x10=100 *actus/acnua*. So the *saltus*, generally signalled on the territory by large roads or roads-plus-canals, were placed at a distance of 10 ETS2 (10x1200 *pes*) or 5 ETS (5x2400 *pes*), 12000 pes, 2400 passus.



The Romulean division of the ETS into 10 parts (ETS2/5): heredium, iugerum, acnua

Let us continue to analyse the series of 10 divisors of ETS and ETS2: 2,3,4,5,6,8,10,12,15,16.
From territorial evidence and the texts of the *agrimensores* we know the decimal division of the ETS, also called *Romulean division* after the foundation of Rome by Romulus described by Livy and other authors. Each square measures 240 *pes*, 48 *passus*, 2 *actus/acnua* and is called *heredium*. Each side of the ETS2 is subdivided into 5 parts.
We have here the proof that the spatial division was not only agrarian but had also been applied to towns (see the project of Rome) since very ancient times, 8 CBCE in Etruria. We find the divisors 5 and 10 used.
If we divide the *heredium* square into two rectangles, its measurements will be 240x120 *pes* or 48x24 *passus,* or 2x1 *actus/acnua*. Thus we have found the rectangle which is the main measurement of both the agrarian and urban decimal spatial division, which was called *iugerum* by the Latin peoples. It is the unitary rectangle of the decimal division of the ETS (or ETS2), measuring 1x2 *actus/acnua*. The subdivision into *iugera* is also attributed by Roman historians to the time of Romulus. The *perfect rectangle* 1x2 is also the basic geometrical figure of the Mesopotamian system of surface measurements.



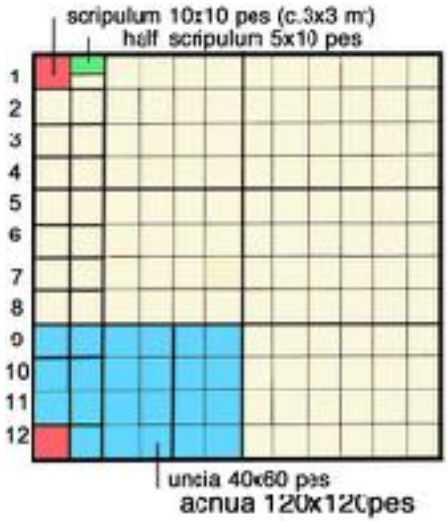
The *iugerum* consists of two *acnua* squares, and the *acnua,* about 36 m per side, is the 1x1 surface unit of the decimal spatial division. To recap, the three surface measurements are multiples of one another and are written as follows:
1 *actus-acnua* square = 120x120 pes = 24x24 *passus* = 1x1 *acnua*
1 *iugerum* rectangle = 240x120 pes = 48x24 *passus* = 2x1 *acnua*
1 *heredium* square = 240x240 pes = 48x48 *passus* = 2x2 *acnua*

(always pairing the measurement with the geometrical figure)

The complete decimal division system of the ETS2

The three measurements identified demonstrate a sequence in the ETS2 decimal division system. Let us insert other surface measurements as submultiples of the *iugerum*: the *uncia*, the *scripulum* and the half-*scripulum*, cited by Iunus Moderatus Columella (1 CACE) in his *"De Iugeribus Metiuendis"*.

I have combined the measurements in an *acnua* square in order to have a schematic division, but should point out that Columella related them to a *iugerum* rectangle. He said that: "in a *iugurum* there are 576 *half-scripula*". The *iugerum (double acnua)* rectangle was the original basis for the ancient spatial division. Hence the *acnua* square was read as a *half-iugerum*.



1 *half-scripulum* rectangle = 5x10 *pes* = 50 *pes*^{2}
1 *scripulum* square = 10x10 *pes* = 100 *pes*^{2}
1 *uncia* rectangle = 40x60 *pes* = 2400 *pes*^{2}

In my onsite analyses, to be precise in the *Portonaccio* Temple of Veii, I found a further measurement of 120/10=12x12 *pes*, which I have called *decimal acnua*. This is perhaps more recent, since I consider the division of the *acnua* by 12 more ancient that that by 10.

Roman literature stops at decimal spatial division, but the numerical systems which we have found show too many references to other classes for us not to hypothesise the existence of further groupings, on 12-6 and 16-8. We have not finished examining the divisors of these numbers.

Further divisors of the ETS-ETS2

Let us continue with the analysis of the use of the divisors of the pair of numbers 2400 and 480. Leaving aside the already-used divisors 2,5,10, the remaining seven numbers 3,4,6,8,12,15,16 clearly appear to be classifiable into two groups: a 3 group (3,6,12,15) and a 4 group (4,8,12,16), with 12 for both series. So far we have seen the use of the classes (2) and (5,10). At least two classes are missing: one based on the divisor 16 (which contains 4 and 8) and one based on the divisor 12 (which contains 3 and 6).

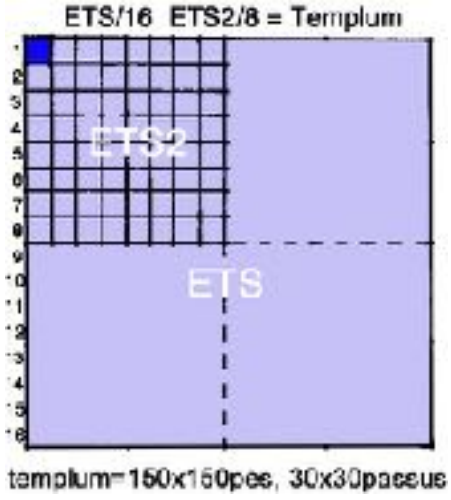
In fact I have found another two divisors of the ETS in the towns and necropolises.

First one according to the divisors 16,8, which generates unitary square modules of 150x150 *pes* (or 150x300 rectangles);

Second one, according to the divisors 12,6, which generates unitary square modules of 200x200 *pes* or 200x400 rectangles.

The divisors 4,8,16 of the ETS-ETS2: the Templum

Let us examine the series 4,8,16. These numbers are very attractive because according to the Latin authors in the Etruscan world they represent the *Templum*, i. e. the *House of the Gods*. The Roman *agrimensores* texts say nothing about them, but in the study of the Etruscan towns many of the town acropolises have shown this spatial division, testifying to the sacred character of areas mainly intended for religious worship.



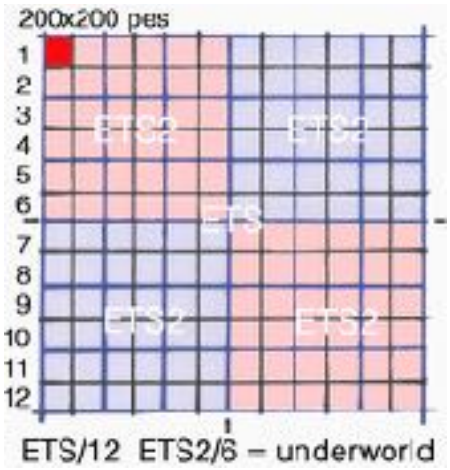
It now seems clear that the choice of the measurement of 2400 *pes* was conceived in order to subdivide the ETS not only into 10x10 heredium squares (the ETS2 in 5x5), but also with other divisions. In this case ETS/16=squares of 150x150 *pes* or 30x30 *passus* equal to approximately 45x45 m (or ETS2/8).

We immediately note that since the measurement of the *acnua* is decimal, it is not compatible with this subdivision. We know neither the Etruscan nor the Roman name of this surface unit. It does exist, because I found it not only in Etruscan towns, but also in the *Ara della Regina* temple in Tarquinia.

This fact means that the different divisions express precise meanings through their measurements. Their numbers might be defined as territorial markers. On this theme, the study of the towns leads me to conclude that the decimal division (10,5) alludes to the space of man, of nature; the hexadecimal one (16,8) instead alludes to the religious space which might be either an acropolis or a temple. It is important to verify whether these measurements coming from the spatial division are used in every context, also the purely architectural one.

The divisors 6,12 of the ETS-ETS2: the underworld

Let us examine the series 3,6,12 and, by analogy with the preceding values let us divide the ETS into 12x12=144 squares of 200x200 *pes* or 40 *passus* (approx. 60 metres) or the ETS2 by 6. The numbers 6 and 12 are the significant ones in this series. We need to observe that the number 6 can be found used in very significant contexts of Etruscan culture: 6 is the measurement of the depth of the Etruscan temple recorded by Vitruvius, and again 6 characterises the base of the labyrinth (cthonium) of Pliny's *fabula* on the *Tomb of Porsenna*. I have found this spatial division in the Etruscan necropolises. A square of 200x200 *pes* (60x60 m) forms the ground-plan of the *Palace of Murlo*, where a cthonian cult is thought to have existed; and the measurement is identical to the Mesopotamian agrarian surface of 1 *iku*. We might indeed call it 1 *Iku* to underline this; it is a proven fact. I also find the possible base rectangle of 200x400 *pes*, an *iku rectangle* (approx. 60x120 m), which again might exist by homology with the *iugerum* one. So essentially, according to the deductions we have made, we might expect to have three geometrical figures of *perfect rectangles* at the base of the spatial division:



the *iugerum rectangle* of 120x240 *pes* (approx. 72x144 m) =1x2 decimal *acnua* = anthropic world
the *templum* r*ectangle* of 150x300 *pes* (approx. 45x90 m) = 1x2 templum *acnua* = celestial world
the *iku rectangle*, of 200x400 *pes* (approx. 60x120 m) = 1x2 *iku acnua* = underworld

Each of these may have had sub-measurements; for example the ground-plan of the *Ara della Regina* temple is 75x150 rather than 150x300.

I conclude this first stage of analysis of the ETS by pointing out the efficacy of the criterion of investigating the numbers according to the classes of their divisors. It has enabled me to make a sure connection with three elements of Latin tradition: on the one hand the *Limitatio* (or *Centuriatio*) of the territory, based on the territorial square of 2400 *pes* per side; on the other hand, two surface measurements mythically attributed to Romulus: the *iugerum* and the *heredium*. Then we have seen that there is not just one Romulean spatial subdivision, but also another two hexadecimal and a duodecimal ones, used to signal one of the three worlds to which they allude: the world of nature and mankind, the earth (5,10), the celestial sphere (8,16); the chthonian or under-world (6,12). We are looking at a *Cosmic Diagram*, where the *Spatial Division* repeats the *Mathematics of the Universe* in a *harmonious* way, linking all the transformations of Mankind together.

At this point, we need to transform the measurements of the *pes, passus, actus*, which we know from Roman tradition, into Etruscan *cubits.*

Introduction to the measurements of the Etruscan cubit

Repeating the actual stages of development of my research, we now introduce the measurement of the Etruscan cubit into this analysis. Remember that the conversion formula from *pes* to *cubit* for the Etruscans is 1x3/5.

The Etruscan's cubit's surprising numbers:

ETS2= 1200x3/5 *pes* = 720x720 cubits (360x360 m)
10x120x3/5 *acnua* = 720x720 cubits
ETS= 2.400x3/5 *pes* = 1.440x1.440 cubits (720x720 m)
20x120x3/5 *acnua* = 1.440x1.440 cubits

In the Spatial Division/5-10: ETS2/5; ETS/10:

1 *acnua* = 72x72 cubits (36x36 m)
1 *iugerum*= 72x144 cubits (36x72 m)
1 *heredium*= 144x144 cubits (72x72 m)

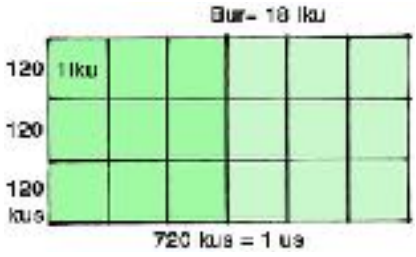
In the Spatial Division/8-16: (*templum*) ETS2/8; ETS/16:
1 *acnua templum*= 90x90 cubits (45x45 m)
1 *iugerum templum*= 90x180 cubits (45x90 m)

In the Spatial Division/6-12: (*iku*) ETS2/6; ETS/12:
1 *acnua iku*= 120x120 cubits (60x60 m)
1 *iugerum iku*= 120x240 cubits (60x120 m)

720 and 1.440 are multiples of 72 (72x10; 72x20).
The basic linear measurements of cubit, *pes* and palm become surface measurements multiplying them by themselves as with the measurement of the side of a square which becomes surface:
1 square cubit (or *pes* or palm) = 1x1

The origin of the Etruscan spatial division

In Etruscan architecture, towns and territory, the system of reference linear measurements consists of the cubit with the *pes* and the *palm*. The surface measurements are expressed by geometrical figures; first and foremost the whole rectangle formed by two squares, and then the square. The importance of analysing the system of the Spatial Division lies in the fact that it contains the whole series of the surface measurements.



Now, after all the references to the *Mathematics of the Origins*, we need to ask ourselves if these measurements are Etruscan/Latin or if they too come from eastern civilisations.
We can affirm that the Etruscan measurements derive from the Mesopotamian linear and surface measurements, and this affirmation justifies the term "*surprising*" that I used earlier. It is in fact surprising to ascertain that they coincide not only as numbers (60, 72, 360, 720 etc.) but also as geometrical figures related to those numbers, starting from the rectangle/double square, which is characteristic of Mesopotamian mathematics.

-the Etruscan cubit, on average 50 cm long, is identical to the Akkadian and later Assyrian cubit (*kus*);

-the measurement of 720 cubits is that of the Mesopotamian *us*;

-the measurement of 120x120 cubits corresponds to that of the Mesopotamian *iku* square;

-the measurement of 720x720 cubits of the ETS2 is the equivalent of 2 Mesopotamian *bur* of 360x720;

-the Roman Centuria ETS of 1440x1440 cubits is made up of 8 *bur* of 360x720 cubits;

-the Mesopotamian *bur* of 360x720 cubits is the basis of the Mesopotamian Spatial Division, then of the Etruscan and Roman ones; in fact the most ancient Etruscan divisions were probably based on the *bur*, as I ascertained for example in the spatial division of the Arno Valley below Fiesole;

-the *iku* square of 120x120 *kus* goes into the *bur* 18 times;

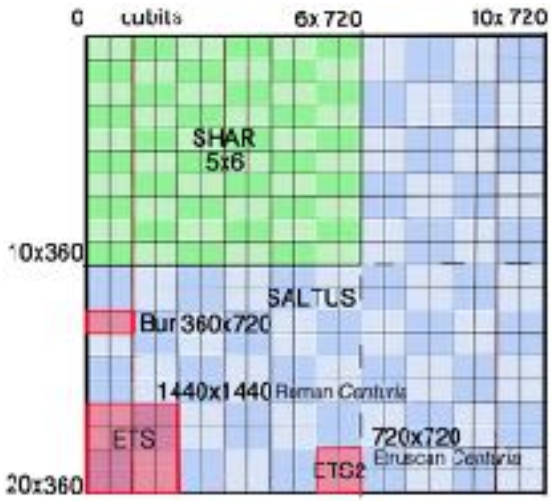
-the *iugerum* rectangle, the basis of the Etruscan/Roman spatial division, goes into the *bur* 25 times. In 1 CACE it is the figure of the double-square rectangle and not of the square which was Columella's reference for the smallest surface measurement, the half- *scripulum* of 3x6 cubits (5x10 *pes*; 1x2 *passus*). This demonstrates its priority over the figure of the square.

-the *shar is* the largest of the Mesopotamian surface measurements, consisting of 10x12 *bur*, for 3600x4320 cubits (1800x2160 m). I never thought I would find this large measurement of 388.8 hectares on the territory, but I did in fact discover it as a planning tool in the three 1MBCE Assyrian capitals and in the territory of Nineveh. Then I found it again at Vulturnum-Santa Maria Capua Vetere, in the planning of Servian Rome, in the territory of the necropolises of Vetulonia; in the division of the Arno plain of Fiesole and Florence; and elsewhere too. It has the proportions 5x6, exactly like the Etruscan temple described by Vitruvius.

-the *half-shar* consists of 5x6 *bur* 1800x2160 cubits (900x1080 m).

-5x6 are the numbers of the Etruscan temple as supplied by Vitruvius; they are also found as measurements of various altars, as for example that of *Portonaccio di Veio*.

5x3 (the same numerical class) are measurements or proportions which distinguish all (and I emphasise all) the central cells of the temples designed according to the Etruscan rule. I have analysed 10 of the most important temples, and 8 have given this result. The other 2, *Temple B* of Pyrgi and temple of *Mater Matuta* in Rome have instead shown a marked Phoenician influence.



-the Latin *saltus* is also homogeneous with these numbers; it is a square with a 2x3600 cubit side, i. e. double the short side of the *shar*, hence 7200x7200 cubits, a square ordered on a decimal basis. It is an evolution of the system which can for the moment be attributed both to the Etruscans (who produced the passage from the *bur* rectangle to the ETS2 square), and to the Romans after 5 CBCE.

In the geometrical figure of the *saltus* I have highlighted the various divisions: *bur*, ETS2 (Etruscan Centuria), ETS (Roman Centuria).
The homogeneity between the Mesopotamian figures (*Bur, Shar*) and the Etruscan and Roman ones can clearly be seen.

-the *Acnua* stands out in the decimal division of the ETS/ETS2 as the main linear surface measurement, 72 and 72x72 Etruscan cubits, respectively. In the analyses of the spatial divisions carried out in the towns, the measurement of 72 cubits (approx. 36 m) emerges as the fundamental measurement of the territory. I should point out that it is not a measurement confined to the Etruscan culture. In my research on the urban planning of the Mediterranean towns of 1 MBCE (9-5 CBCE), including also the Greek colonial towns and the Anatolian ones, the most frequent measurement in the width of the urban blocks oscillates around 35-37 m (depending on the units of measurement), thus revealing the presence of a widespread international town-planning culture of Assyrian origin. In the African Greek colony of Cyrene I have found the *iugerum* (36x72 m about). The Etruscan culture appears to be substantially canonical in relation to the sources, unlike the Greek one.

Square or rectangle?

The fundamental importance of the rectangle/double square in the Mesopotamian spatial division is evident, as is its influence on Etruscan culture. Now the question may arise as to whether my initial affirmations on the characteristics of the ETS and ETS2 squares are still valid.
My opinion, backed up by analyses of the architecture, towns and territories also in Egypt and Mesopotamia, is that the Etruscan *Project Culture* presents elements of both these. Egyptian land and urban geometrical division is based on squares. While the Etruscan Spatial Division assumes initially the Mesopotamian *bur* and afterwords the ETS2 square as *double-bur*, the architecture of the *Tumuli* show a complex numerical basis of Egyptian origin, which mathematically defines the basic relationships between the square and the circle. So, we are looking at mixtures evidently deriving from the closeness of the two cultures. It should be pointed out that the Etruscan and Mycenaean *tumuli,* though diachronic, have exactly the same mathematical language, to the extent of being interchangeable. In Mycenae I found the Mycenaean cubit, with the same mathematical characteristics as the E*gyptian royal cubit*, i. e. its division into 7 *palms*. The square, not the rectangle/double square, is at the basis of Egyptian surface measurements and spatial

division. Nonetheless, the Chamber of the King in the Khufu Pyramid is a rectangle/double square, thus underlining the similarities between Egypt and Mesopotamia.
The choice of the square ETS2 (720x720 cubits), instead of the *bur* rectangle (360x720 cubits) appears to me an evolutionary feature of the Etruscan Project Culture.

Comparative tables of the Mesopotamian, Etruscan and Roman surface measurements

	meas.ts	Mesopotamian	meas.ts	Etruscan	meas.ts	Roman
measurements	**base: cubit kus=1**	**medium value cm 50,00**	**base: cubit=1**	**medium value cm 50,00**	**base: foot= 1**	**real value cm 29,65 30,00 to compare**
cubit²	**1x2**	**50x100**	**1x1**	**50x50**		
foot-pes²			**6/10x6/10**	**30x30**	**1**	**29,65x29,65**
palm²			**1/10x1/10**	**5x5**		
Mesopotamian "brick"	**1/2x2/5**	**25x20**				
	cubits	**metres**	**cubits**	**metres**	**feet**	**metres**
1/2 scripulum			**6x3**	**3x1,5**		
scripulum/ pertica²			**6x6**	**3x3**	**10x10**	**3x3**
sar	**12x12**	**6x6**	**(12x12)**	**(6x6)**		
uncia			**24x36**	**12x18**		
verga					**60x60**	**18x18**
actus/acnua	**(72x72)**	**(36x36)**	**72X72**	**36x36**	**120x120**	**36x36**
iugerum	**(72x144)**	**(36X72)**	**72x144**	**36X72**	**120x240**	**36X72**
luger. templum			**90x180**	**45x90**		
luger. ctonio			**120x240**	**60x120**		
iku	**120x120**	**60x60**	**120x120**	**60x60**		
heredium	**(144x144)**	**(72x72)**	**144X144**	**72x72**	**240x240**	**72x72**
bur	**360x720**	**180x360**	**360x720**	**180x360**		
double bur/ Etruscan centuria	**720x720**	**360x360**	**720x720**	**360x360**		
Roman centuria		**(8 bur)**	**1440x1440**	**720x720**	**2400x2400**	**720x720**
shar	**3600x4320**	**1800x2160**	**3600x4320**	**1800x2160**		
1/2 shar	**1800x2160**	**900x1080**	**1800x2160**	**900x1080**		
saltus			**7200x7200**	**3600x3600**	**12.000x12.000**	**3600x3600**

Extract from: *The search for E, The Project of Architecture, Town and Territory in Etruria and the Ancient World* (to be edited). www.mariopreti.it

NOTES
RC = Royal Cubit RF = Royal Foot GC = Etruscan Cubit (Golden Cubit)
C = Century M = Millennium BCE = Before Common Era ACE = After Common Era
MBCE = Millennium Before Common Era CACE = Century Before Common Era